

高強度鋼筋混凝土梁之 裂縫發展與控制

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Life-Cycle of
Structure Engineering

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Content

COURSE

Destination

Design Codes

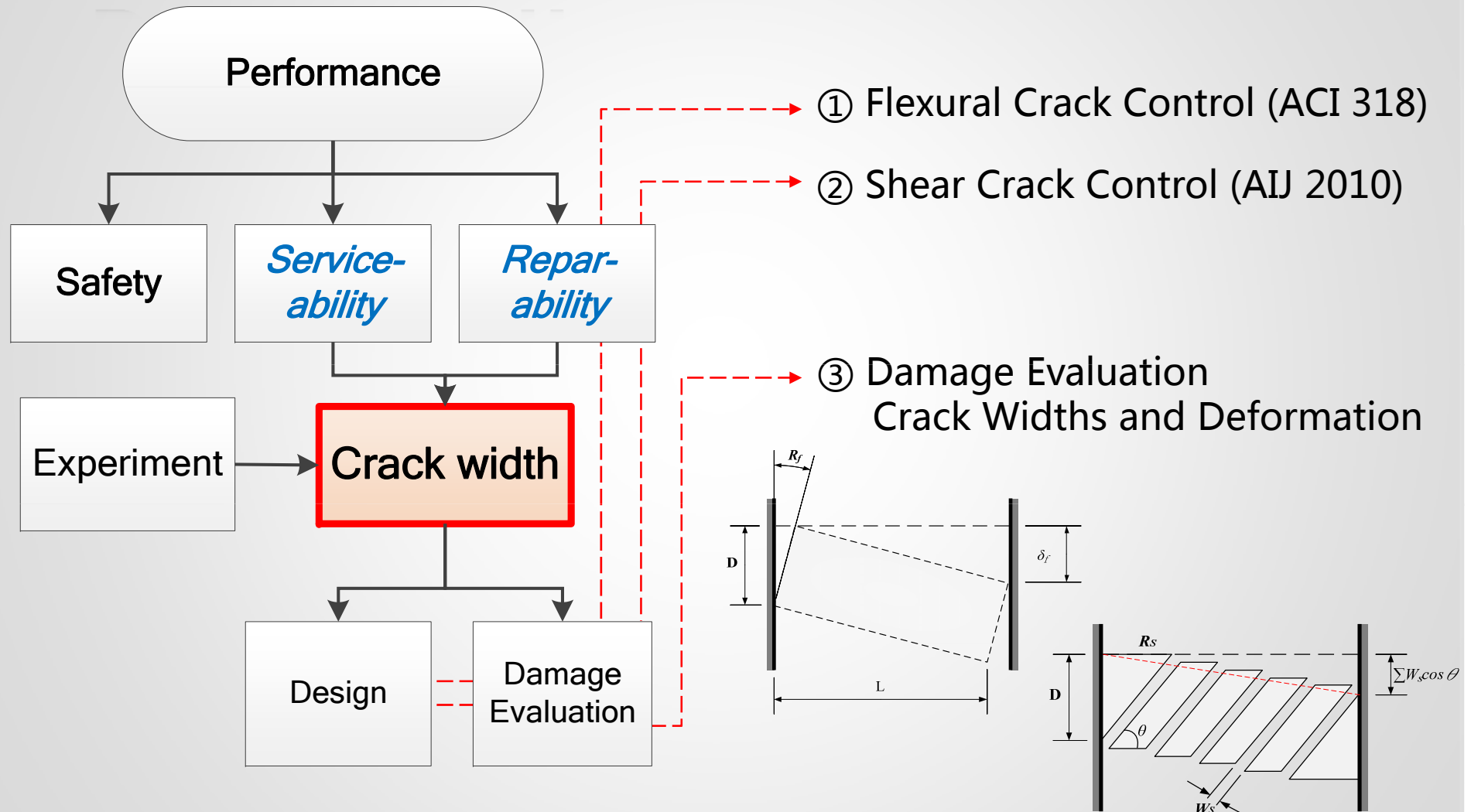
Experiment

Shear Crack Control

Flexural Crack Control

Conclusions

Destination



Shear Crack Control

AIJ (2010)

Allowable shear force corresponding to serviceability ensuring

$$V_{AL1} = bj\alpha f_{cs}$$

$$V_{AL1} = bj\{\alpha f_{cs} + 0.5f_{ss}(p_w - 0.002)\}$$

Allowable shear force corresponding to reparability ensuring

$$V_{SL1} = bj\left\{\frac{2}{3}\alpha f_c + 0.5f_s(p_w - 0.002)\right\}$$

Reinforcement

Types	Long-term allowable stress		Short-term allowable stress	
	Compression and tension	Shear	Compression and tension	Shear
SR 235	155	155	235	235
SR 295	155	195	295	295
SD 295				
SD 345	195	195	295	295
SD 390	215 (*195)	195	345	345
SD 490	215 (*195)	195	390	390
			490	490

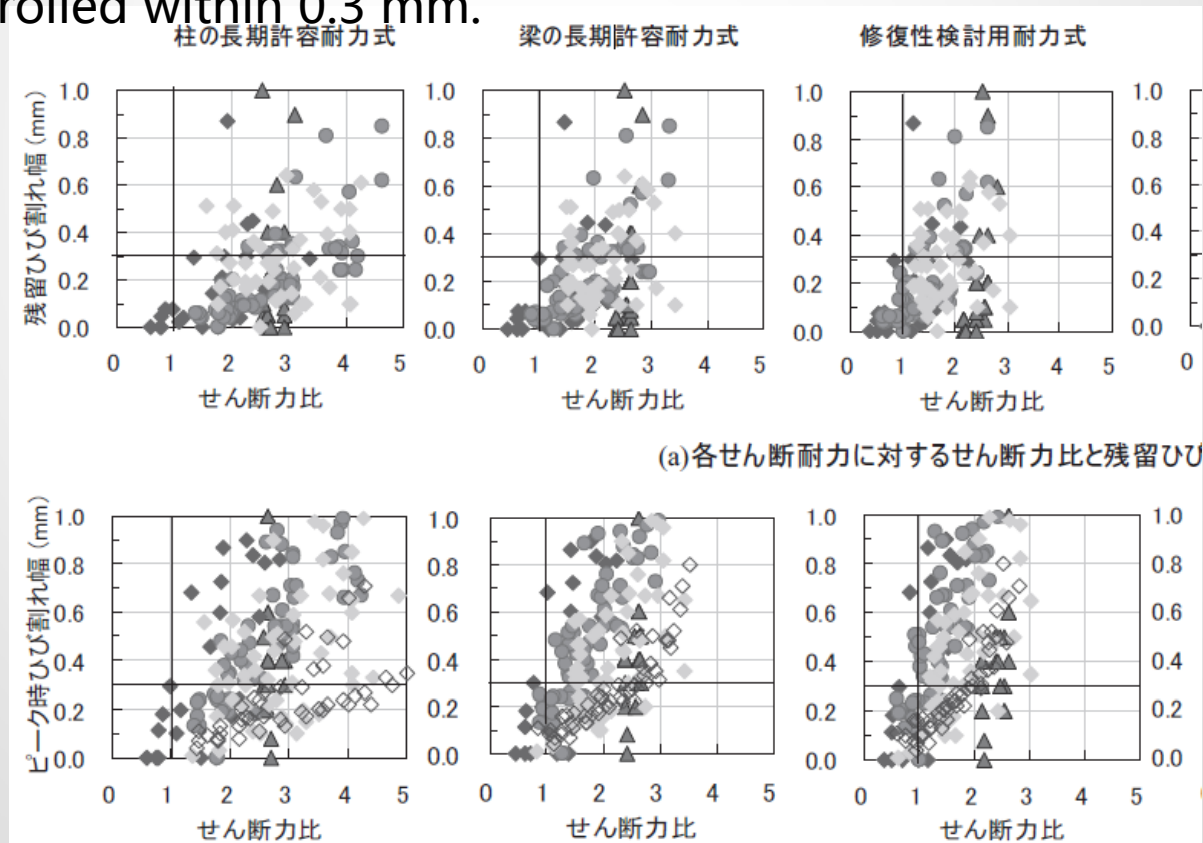
Concrete

Long-term allowable stress		Short-term allowable stress	
Compression	Shear	Compression	Shear
$\frac{1}{3}f'_c$	$\min\left\{\frac{1}{30}f'_c\left(0.49+\frac{1}{100}f'_c\right)\right\}$	Long-term allowable stress $\times 2.0$	Long-term allowable stress $\times 1.5$

Shear Crack Control

AIJ (2010)

According to the research conducted by Simazaki (2009), if the long-term loading is less than V_{AL1} , the *maximum shear crack width* can be controlled within 0.3 mm. If the short-term loading of a medium-magnitude is less than V_{SL1} , the *residual maximum shear crack width* can be controlled within 0.3 mm.



Flexural Crack Control

ACI-318 (2005)

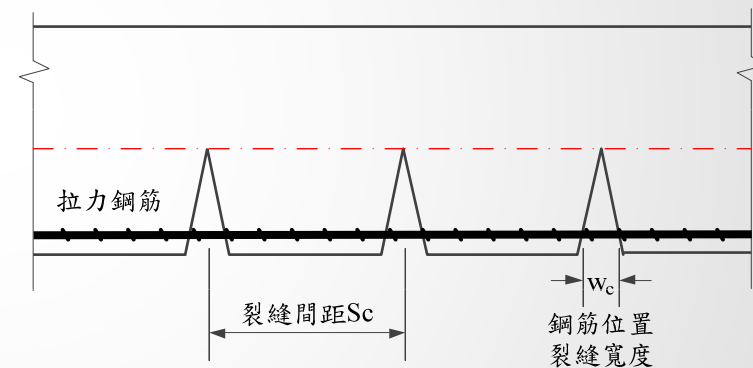
$$s \leq 380 \left(\frac{280}{f_s} \right) - 2.5c_c$$

$$s \leq 300 \left(\frac{280}{f_s} \right)$$

Ref. Robert J. Frosch (1999)

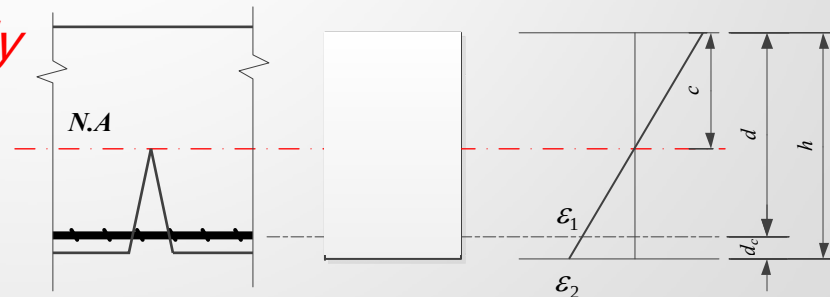
(1) Model:

$$w_c = \varepsilon_s \cdot S_c = \frac{f_s}{E_s} \cdot S_c$$



Crack width grows linearly

$$w_{max} = \frac{h - c}{d - c} \cdot w_c = \beta \cdot w_c$$



Flexural Crack Control

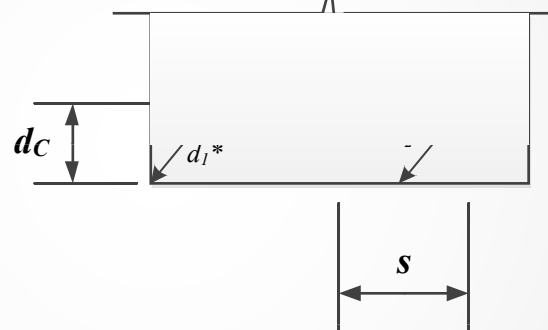
ACI-318 (2002 & 2005)

Ref. Robert J. Frosch

(1999)
(2) Crack spacing

From the bar center to the farthest
concrete element

$$w_{max} = \frac{h - c}{d - c} \cdot \frac{f_s}{E_s} \cdot S_c$$



$$d_1^* = \sqrt{d_c^2 + d_c^2}$$

$$d_2^* = \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

Control !

Minimum crack spacing = $1.0 \times d_2^*$

Average crack spacing = $1.5 \times d_2^*$ → $w_{max} = \beta \cdot \frac{f_s}{E_s} \cdot 2 \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$

Maximum crack spacing = $2.0 \times d_2^*$

Flexural Crack Control

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ACI-318 (2002 & 2005)

$$s = 380 \frac{250}{f_s} - 2.5C_c \leq \frac{300 \cdot 252}{f_s} \quad 0.41 \text{ mm}$$

Ref. Robert J. Frosch (1999) $s = 380 \frac{280}{f_s} - 2.5C_c \leq \frac{300 \cdot 280}{f_s} \quad 0.41-0.53 \text{ mm}$

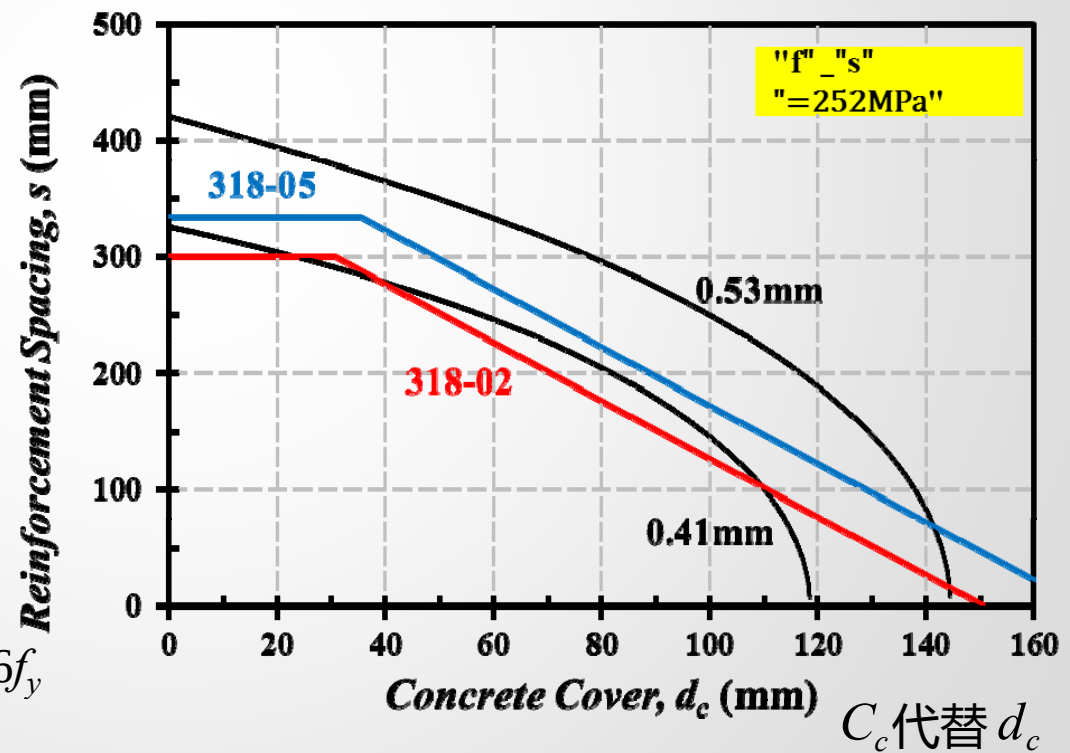
(3) Crack control

$$w_{max} = \beta \cdot \frac{f_s}{E_s} \cdot 2\sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

➔ $s = 2\sqrt{\left(\frac{w_{max} E_s}{2\beta f_s}\right)^2 - d_c^2}$

Crack width : 0.41 mm

Tensile stress of main bars : $0.6f_y$



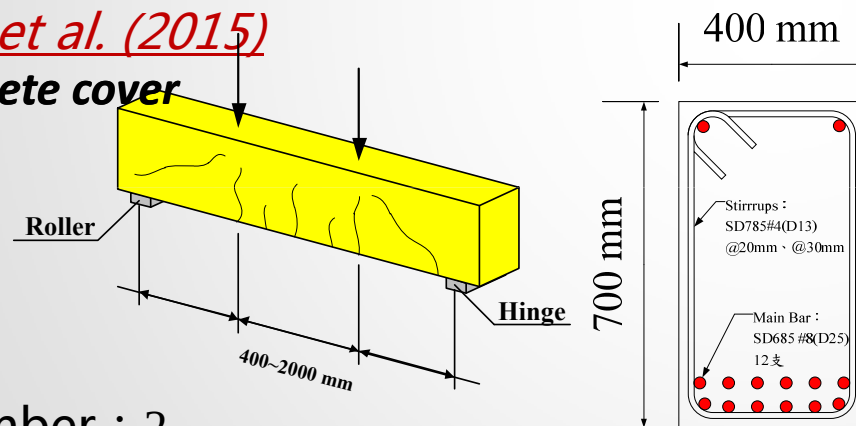
Experiment

Specimen Design

Yielding stress	685、785MPa
Reinforcement size	D32(#10)、D25(#8)
Compressive strength	70、100MPa
Section size	70×40、50×35 (cm ²)

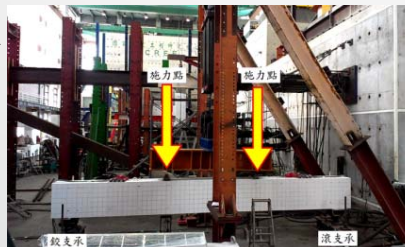
Chiu et al. (2015)

Concrete cover



Number : 2

Concrete cover : 2、3cm

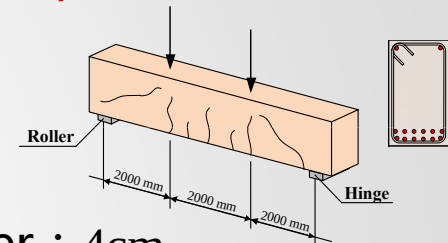


Chiu et al. (2013)

Stirrup ratio

Number : 10

Concrete cover : 4cm

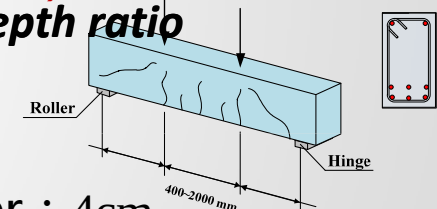


Chiu et al. (2014)

Shear span to depth ratio

Number : 10

Concrete cover : 4cm

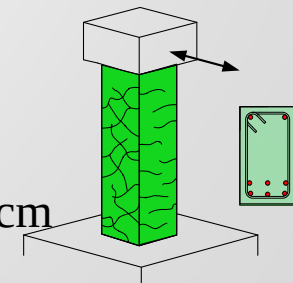


Cheng et al. (2014)

Cyclic loading

Number : 3

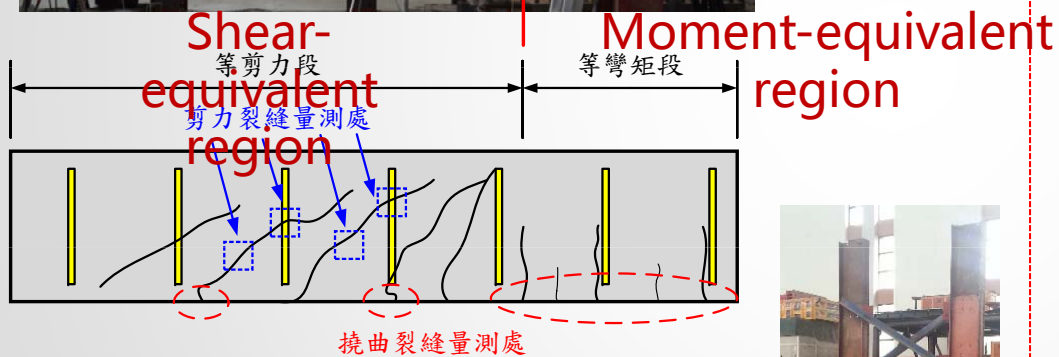
Concrete cover : 3cm



Experiment

Measurement system

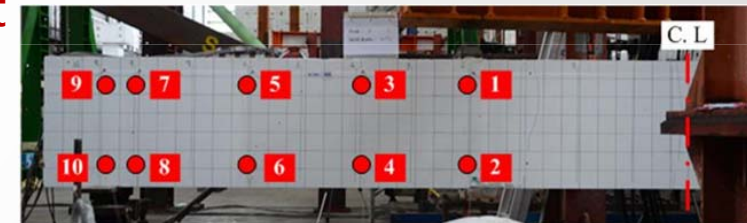
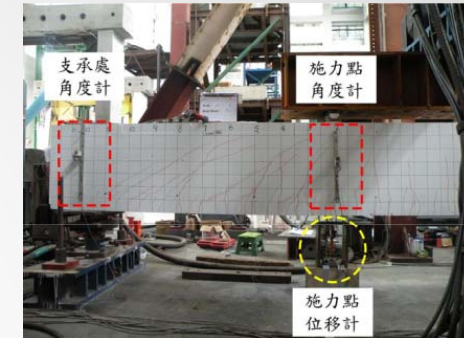
1. Crack measurement



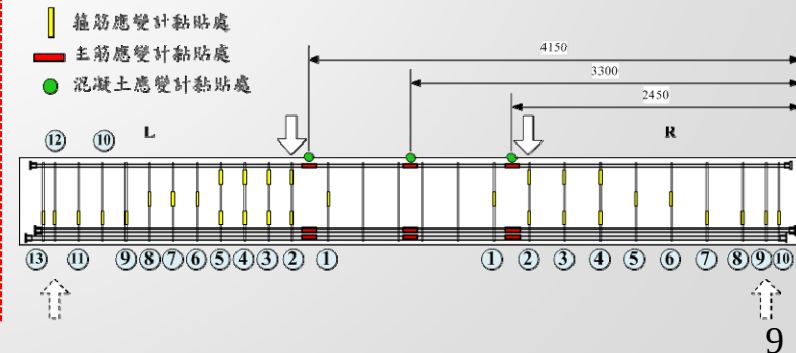
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2. NDI



3. Strain of reinforcement

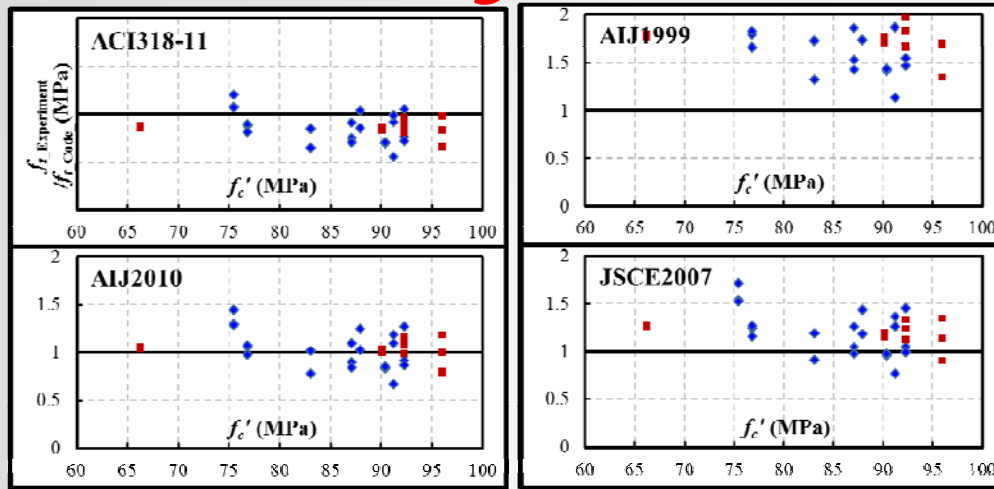


Experimental Data

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Shear Crack

Tensile Strength of Concrete

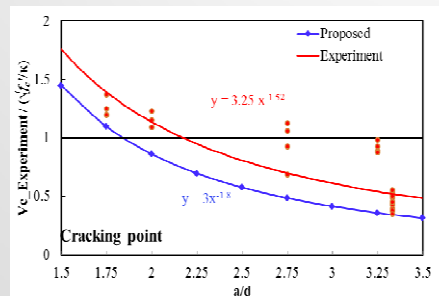


AIJ1999	$0.33\sqrt{f'_c}$	MPa
AIJ2010	$0.56\sqrt{f'_c}$	MPa
JSCE2007	$0.23f'_c{}^{2/3}$	MPa

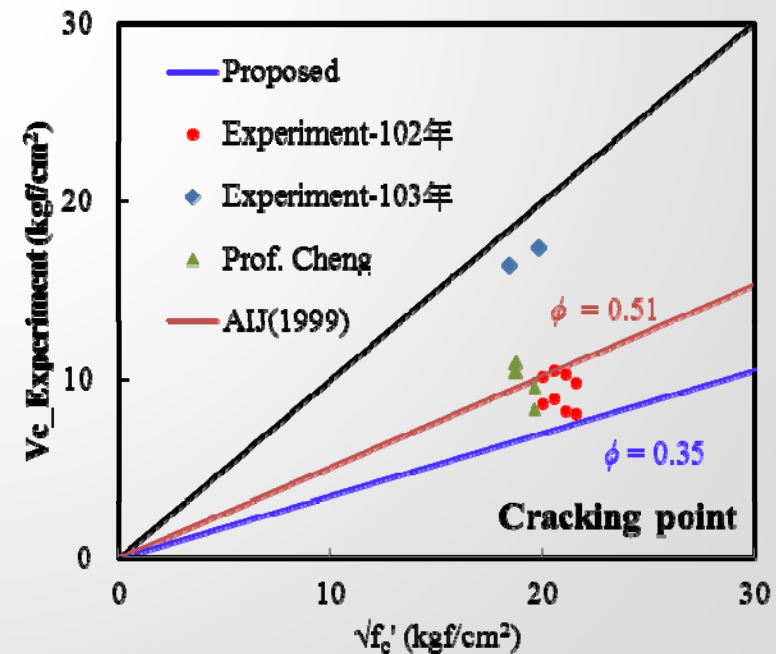
Shear Crack Strength of Concrete

$$v_{cr} = \eta_{cr} \times 0.33\sqrt{f'_c} / \kappa \text{ (MPa)}$$

$$\eta_{cr} = 3 \left(\frac{a}{d} \right)^{-1.8} \quad (0.35 \leq \eta_{cr} \leq 1.0)$$



LCSE



Experimental Data

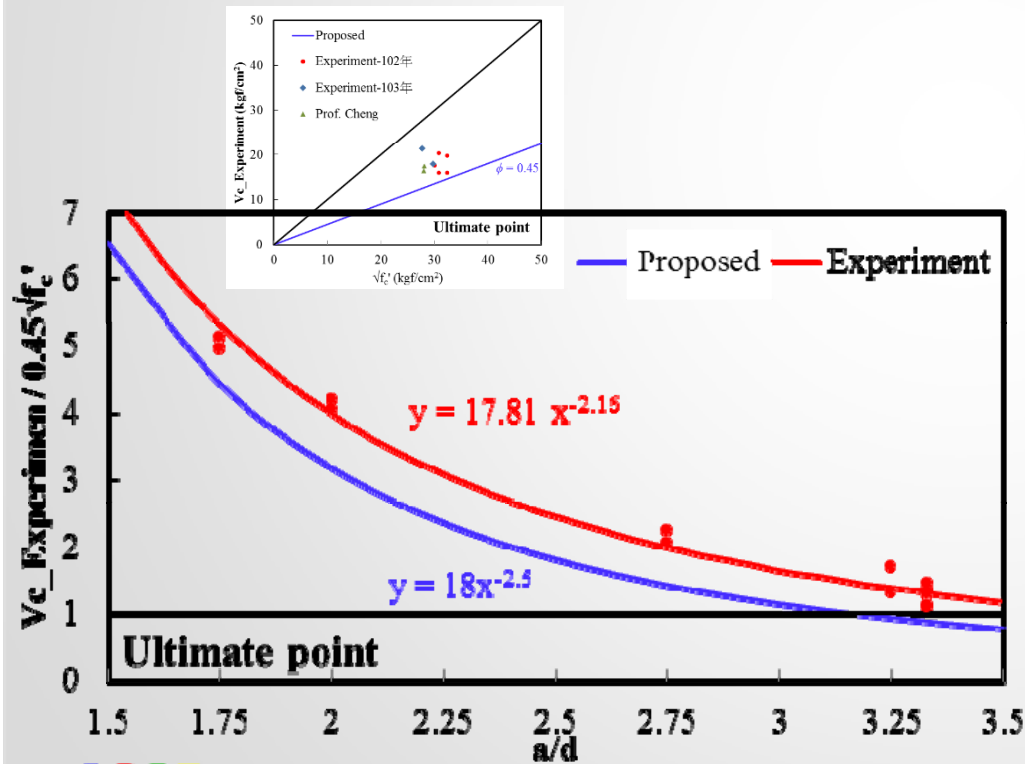
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Shear Crack

Ultimate Shear Strength of Concrete

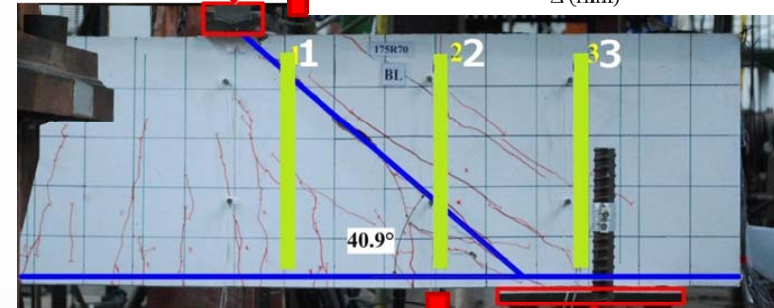
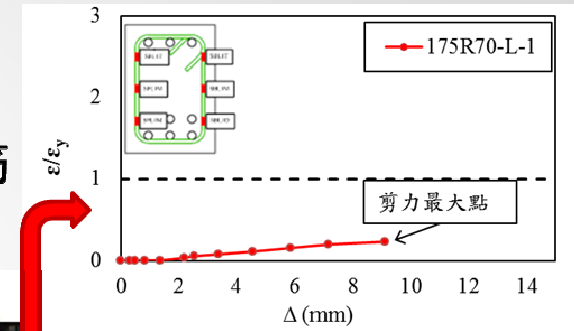
$$v_{cu} = 0.45\eta_{cu} \times 0.33\sqrt{f'_c} \text{ (MPa)}$$

$$\eta_{cu} = 18 \left(\frac{a}{d}\right)^{-2.5}; (1 \leq \eta_{cu} \leq 4) \text{ (MPa)}$$

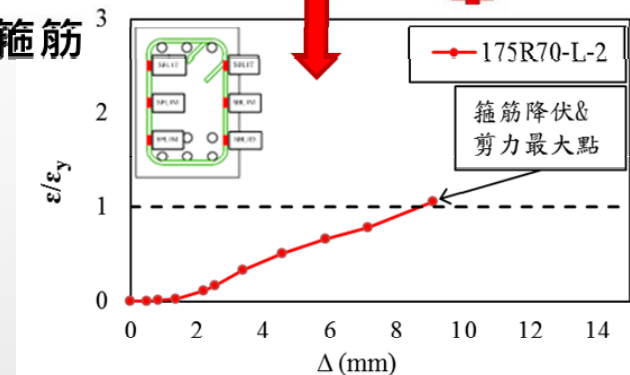


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1號箍筋



2號箍筋



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Experimental Data

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Shear Crack

$$v_{c_ser} = v_{cr}$$

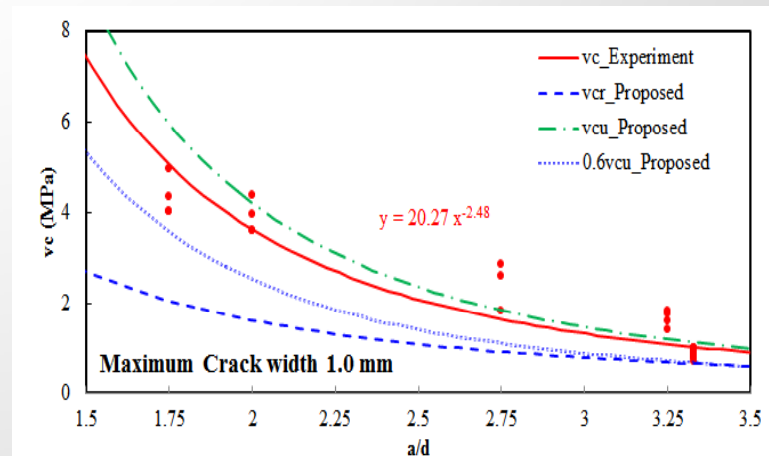
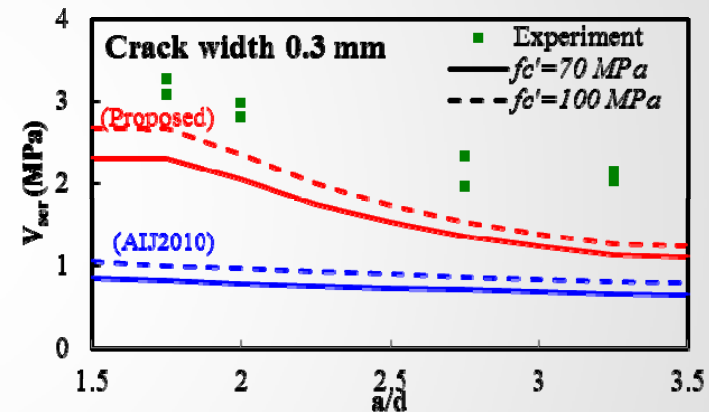
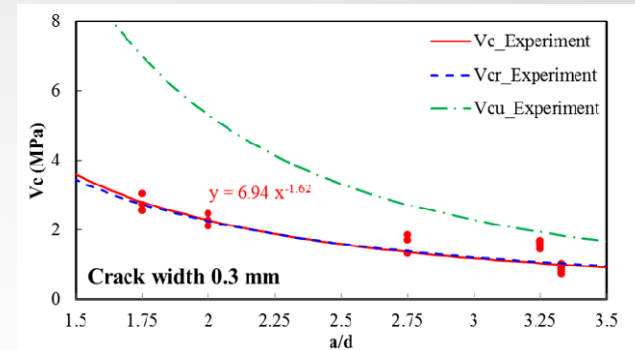
$$v_{cr} = \eta_{cr} \times 0.33 \sqrt{f'_c} / \kappa \text{ (MPa)}$$

$$\eta_{cr} = 3 \left(\frac{a}{d} \right)^{-1.8} \quad (0.35 \leq \eta_{cr} \leq 1.0)$$

$$v_{c_rep} = 0.6 v_{cu}$$

$$v_{cu} = 0.45 \eta_{cu} \times 0.33 \sqrt{f'_c} \text{ (MPa)}$$

$$\eta_{cu} = 18 \left(\frac{a}{d} \right)^{-2.5} ; (1 \leq \eta_{cu} \leq 4) \text{ (MPa)}$$



Shear Crack Control

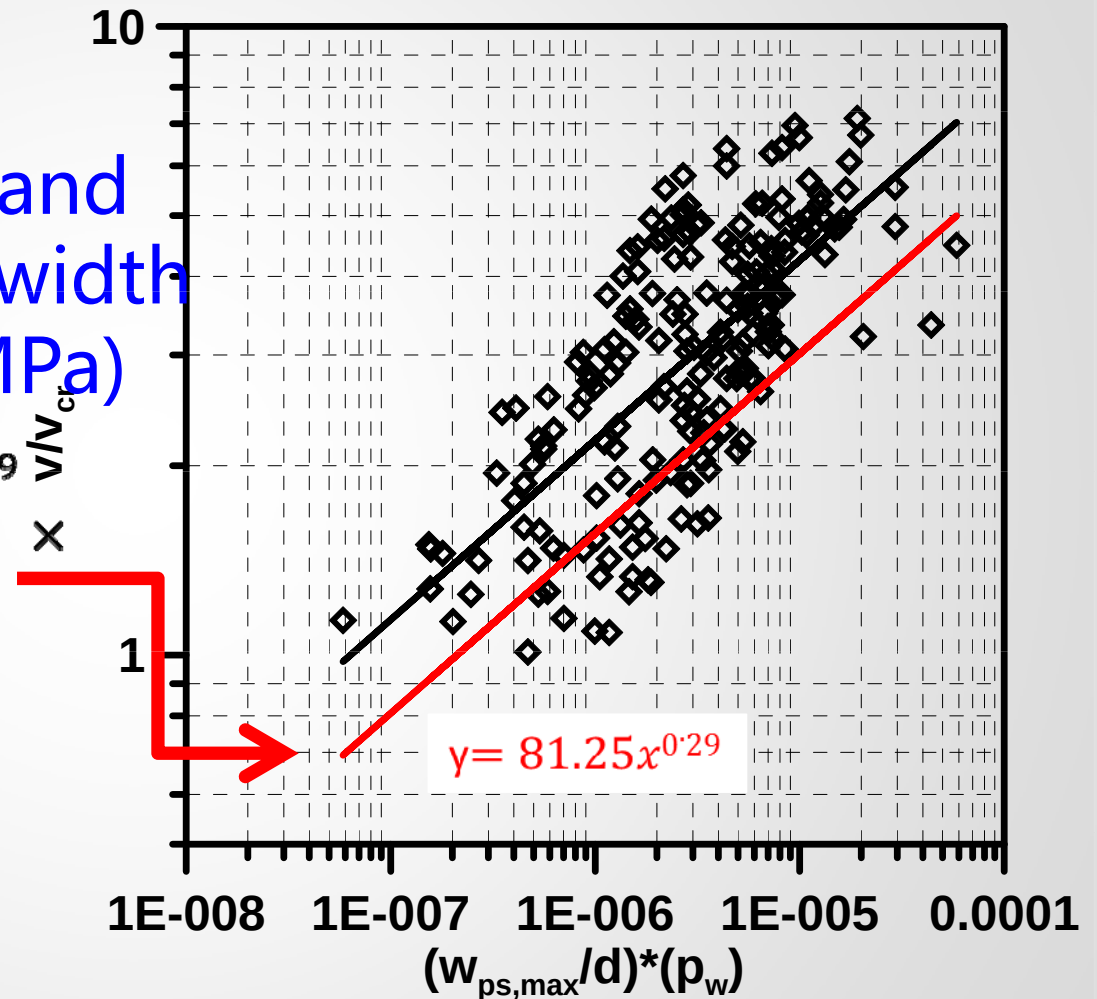
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Design Formula

Average shear stress and
Maximum Shear crack width
(Reinforcement 785MPa)

$$V = 81.25 \times \left\{ \left(\frac{W_{ps,max}}{d} \right) \times \rho_{ts} \right\}^{0.29} \times \left(\eta_{cr} \times (0.33\sqrt{f'_c}) \times bD/1.5 \right)$$

$$\rho_{ts} = \frac{A_v}{bd}$$



Experimental Data

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Shear Crack

Allowable Shear Stress

(Maximum shear crack width 0.3 mm)

$$V_c = \frac{(V_{u,1mm} - V_{sc})}{(1.0 - 0.3)} \times (W_{p,max} - 0.3) + V_{sc}^{1.2}$$

$$V_s = V - V_c = \rho_{ts} f_s b d$$

$$f_{st} = \frac{V - V_c}{\rho_{ts} b d}$$

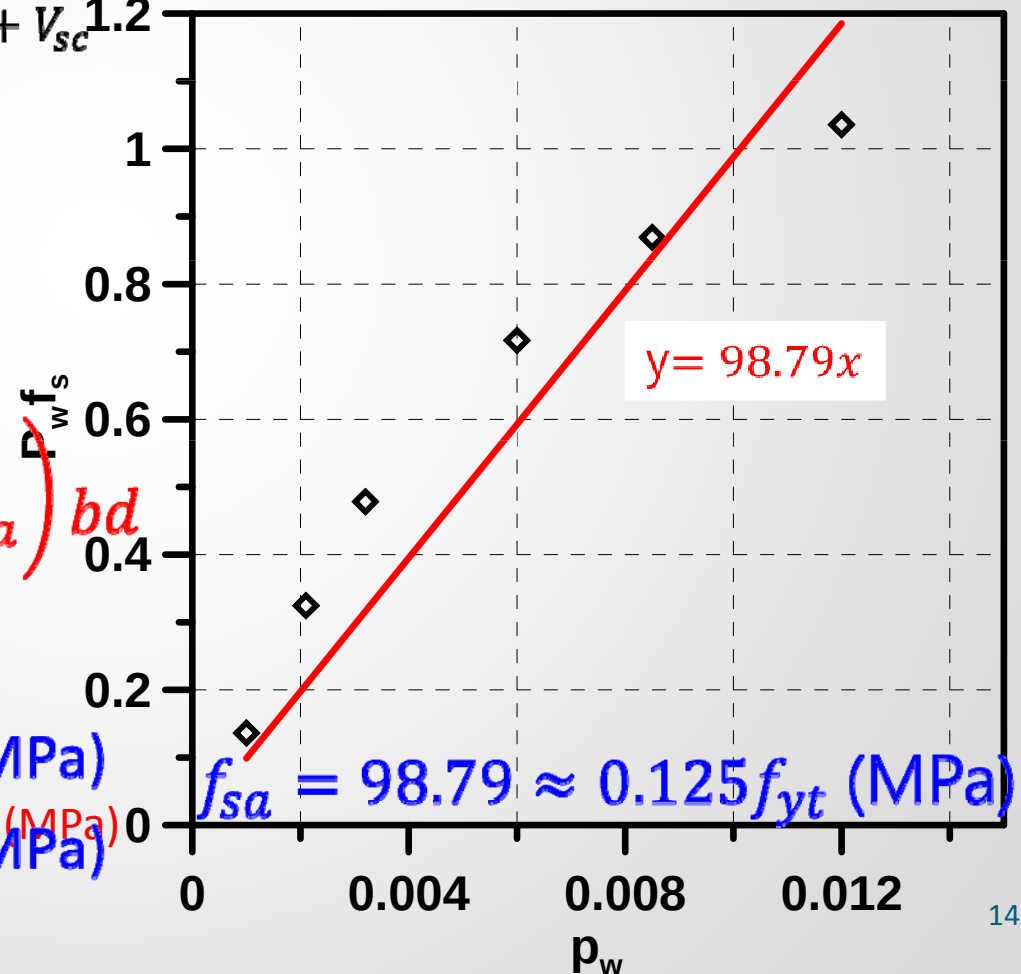
$$V_{sa} = \left(\frac{\eta_{cr} \times 0.33 \sqrt{f'_c}}{1.5} + \rho_{ts} f_{sa} \right) b d$$

$$\eta_{cr} = 3 \times \left(a/d \right)^{-1.8}; 0.35 \leq \phi \leq 1.0$$

$$f_{sa} (0.3 \text{ mm}) = 0.125 f_{yt} \text{ (MPa)}$$

$$f_{sa} (0.4 \text{ mm}) = 0.150 f_{yt} \text{ (MPa)}$$

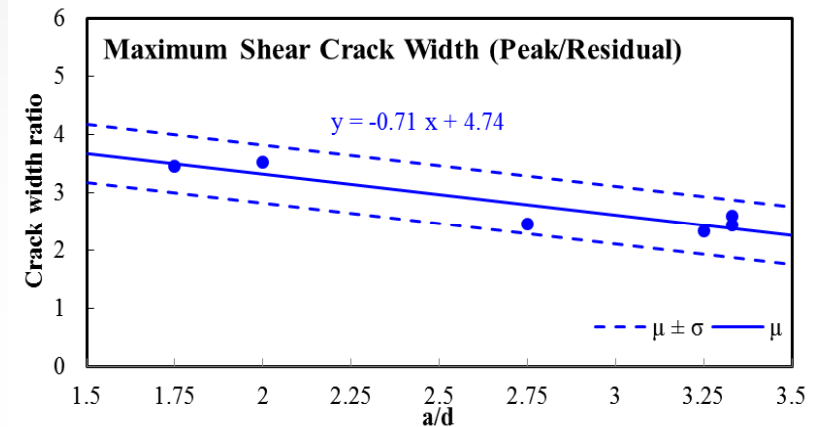
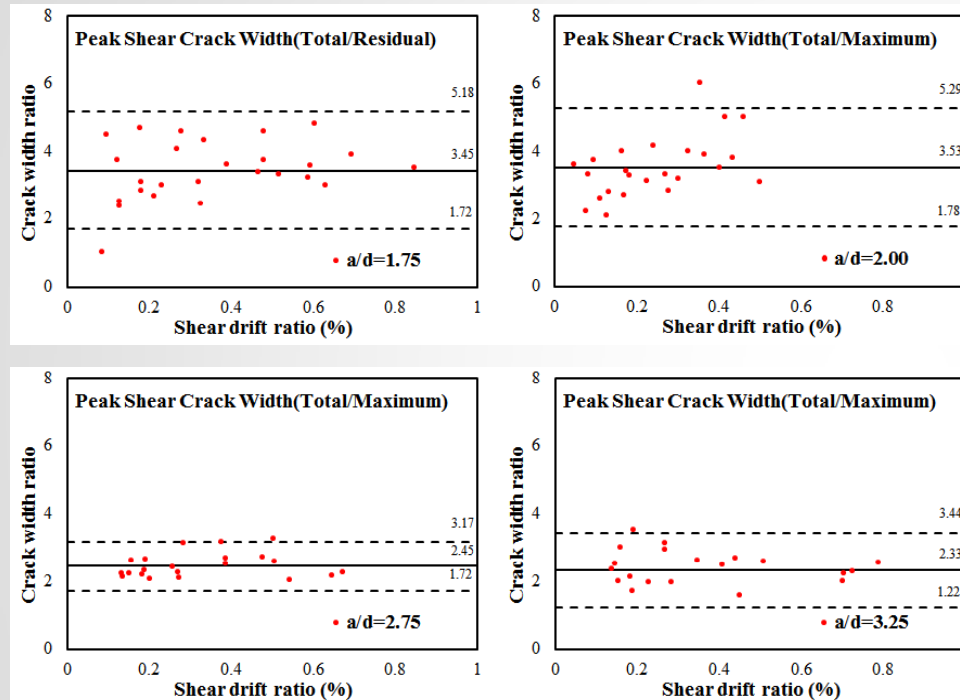
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Experimental Data

Shear Crack

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$$n_{s_Maximum} = -0.71 \times \left(\frac{a}{d}\right) + 4.74$$

Allowable Shear Stress
(Maximum shear crack width 1.0 mm)

$$f_{sa} (1.0 \text{ mm}) = 0.20f_{yt} \text{ (MPa)}$$

Allowable Shear Stress
(Maximum residual
shear crack width 0.4
mm)

Shear Crack Control

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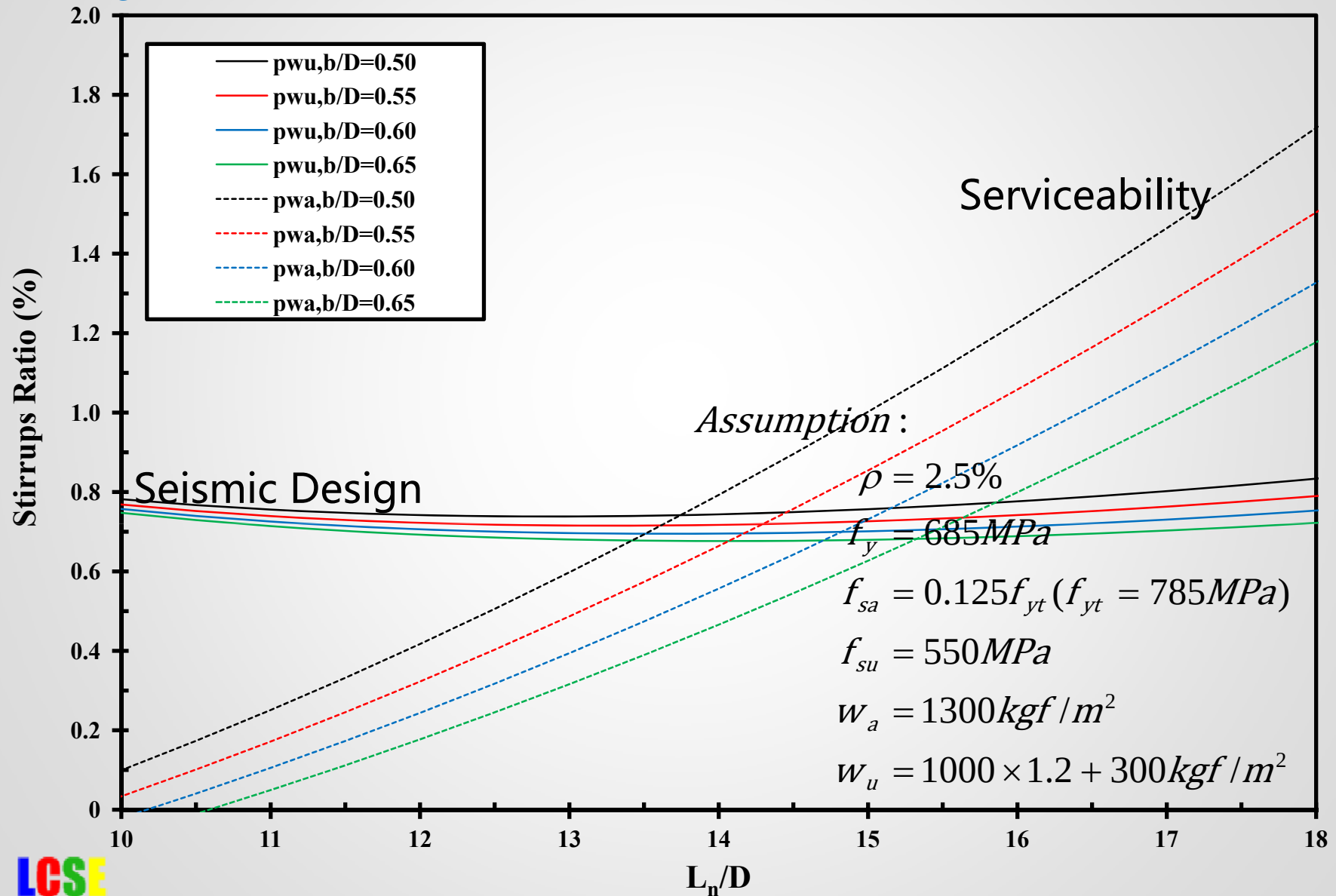
Design Formula

Performance			Allowable shear stress (MPa)
Long - term	Serviceability	Shear crack strength	$v_{cr} = \eta_{cr} \times 0.33\sqrt{f'_c}/1.5$ $\eta_{cr} = 3 \left(\frac{a}{d}\right)^{-1.8} \quad (0.35 \leq \eta_{cr} \leq 1.0)$
		Maximum shear crack width 0.4mm	$v_{ser} = \eta_{cr} \times 0.33\sqrt{f'_c}/1.5 + 0.15\rho_{ts}f_{yt}$
Short - term	Reparability	Maximum shear crack width 1.0 mm	v_{rep} $= 0.27\eta_{cu} \times 0.33\sqrt{f'_c} + 0.20\rho_{ts}f_{yt}$ $\eta_{cu} = 18 \left(\frac{a}{d}\right)^{-2.5} ; (1 \leq \eta_{cu} \leq 4)$

Shear Crack Control

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Design Formula



Shear Crack Control

Design Formula

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Random variables

b/D: 0.45-0.55

B/L: 0.5-1.0

Dead Load : 800-1100 kgf/m

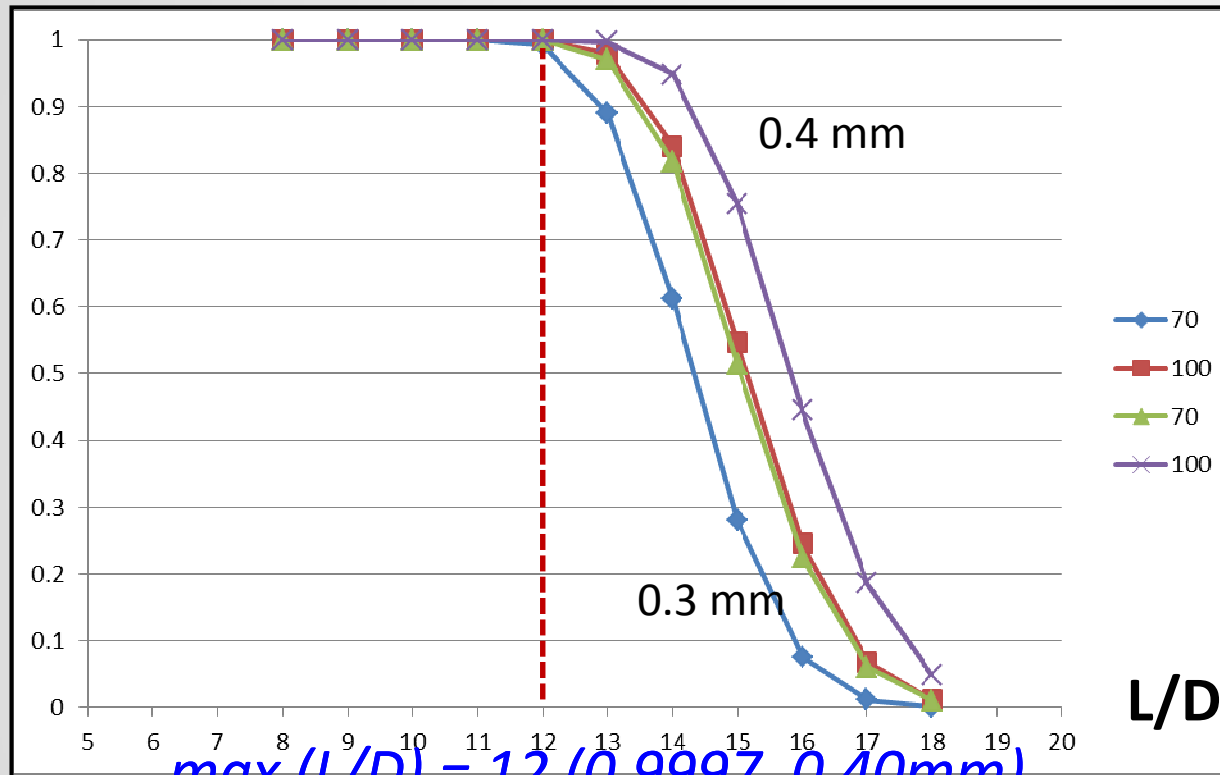
Live Load: 200-400 kgf/m²

f_y: 685 Mpa

f_{yt}: 785 Mpa

f_{su}: 550 MPa

f_{sa}: 0.125 or 0.15 f_{yt}



max (L/D) = 12 (0.9997, 0.40mm)

$$\rho_{ts,u} = \left(C_1 / \left(\frac{L}{D} \right) + C_2 \left(\frac{L}{D} \right)^2 \right) / f_{su}$$

$$\rho_{ts,a} = \left(C_3 \left(\frac{L}{D} \right)^2 - v_{cr} \right) / f_{sa}$$

$$G(L/D) = \frac{\rho_{ts,u}}{\rho_{ts,a}} - 1 \Rightarrow \text{Reliability}$$

MCS

Flexural Crack Control

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手冊建議, 4.3.2

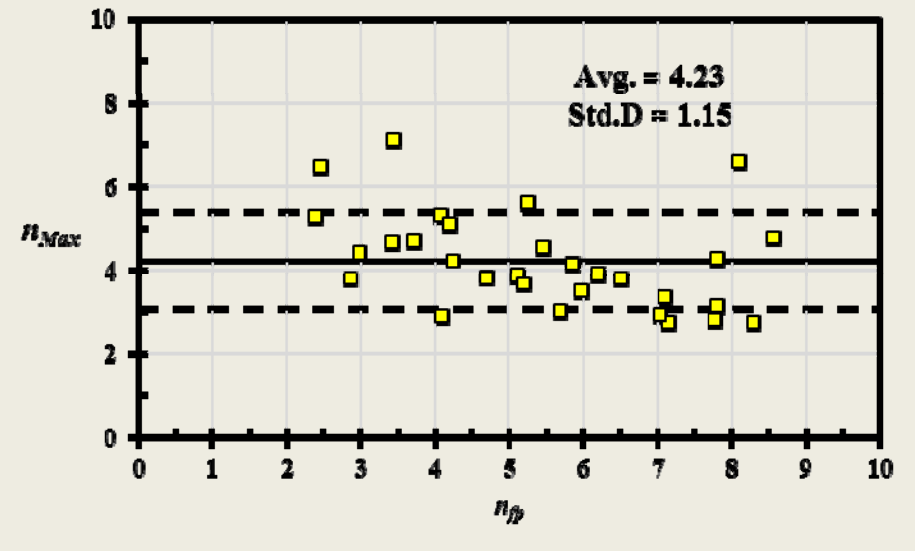
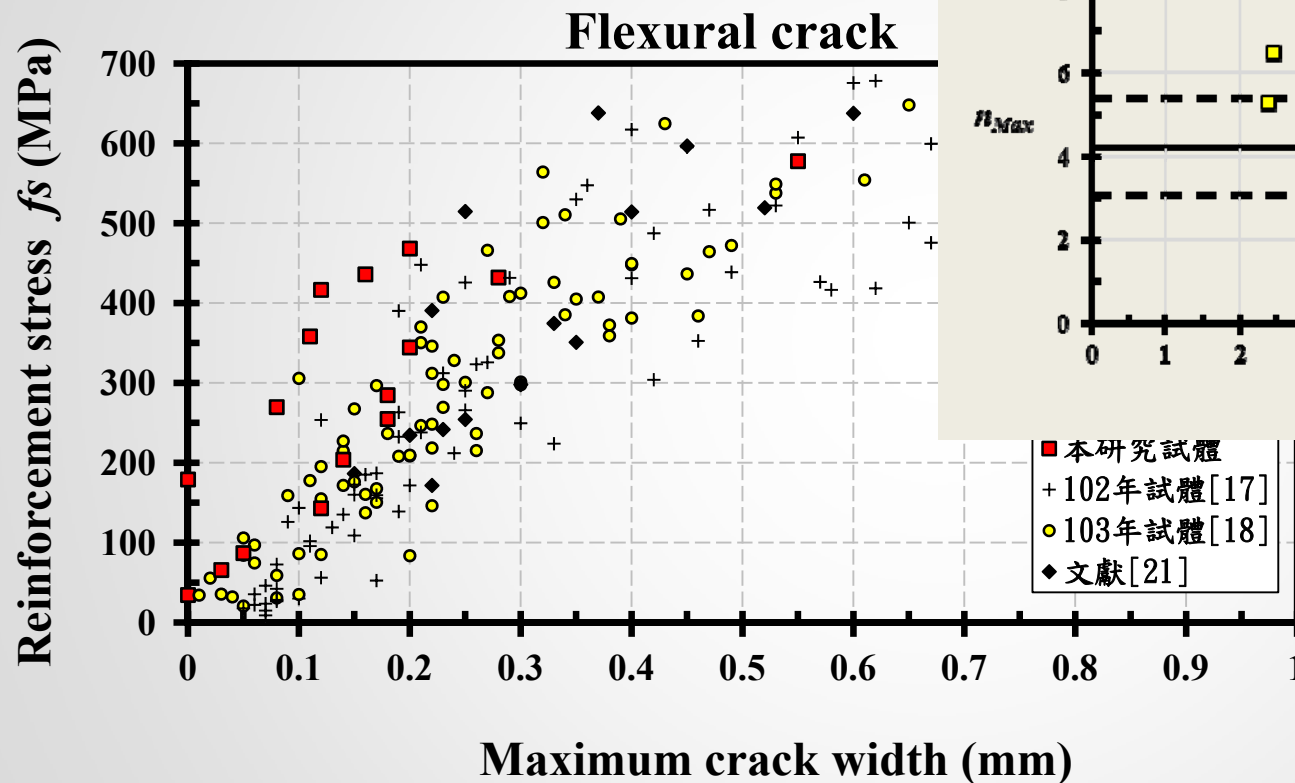
$$v_{ser} = \frac{0.35\sqrt{f'_c}}{1.5} + \frac{A_v f_s}{b_w s} \qquad v_{ser} = \frac{0.35 \times 0.33\sqrt{f'_c}}{1.5} + \frac{A_v f_s}{b_w s}$$

f_s 為剪力鋼筋之容許應力，最大值為 $0.15f_{yt}$

[解說] 設計剪應力之計算可參考ACI318之建議， $v=V/(b_w d)$ 。計算梁構件於靜載重與活載重下之斷面剪應力時，不須考慮兩載重之載重放大因子。式(4.3.1)之前項為混凝土之容許應力，依實驗結果可知，其與構件之剪力跨距與深度比有密切相關，就一般中長梁而言則建議採用0.35[3]。由式(4.3.1)可知，梁構件可藉由剪力鋼筋之用量以提升其容許應力。若梁構件之跨深比增加，為有效控制靜載重與活載重下之剪力裂縫，則具所需之剪力鋼筋用量亦會增加，並會大於耐震設計下剪力鋼筋量；反之，若能將梁構件之跨深比降至12以下，則耐震設計下剪力鋼筋需求會主控設計，而不須檢核本節規定[3]。

Experimental Data

Flexural Crack



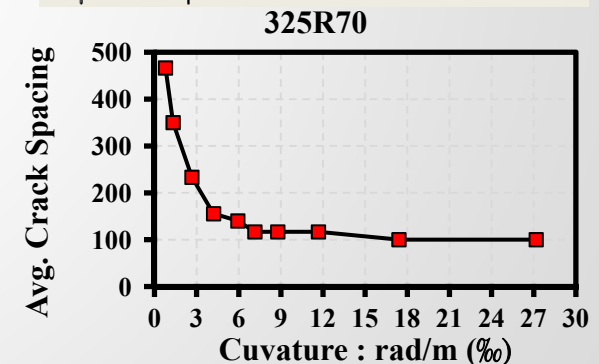
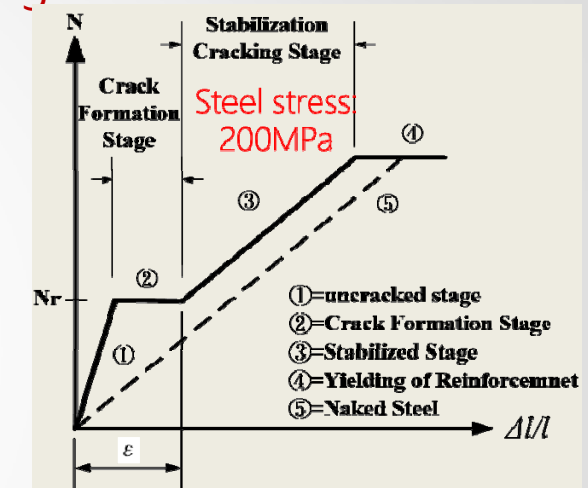
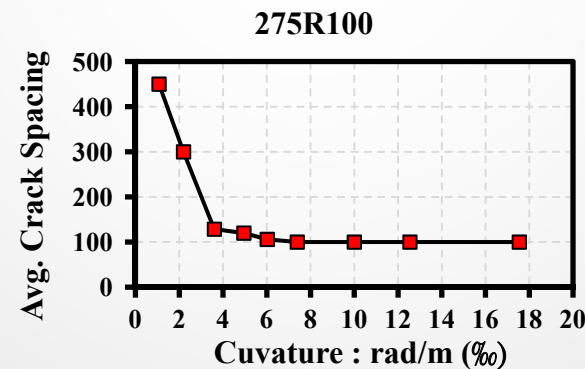
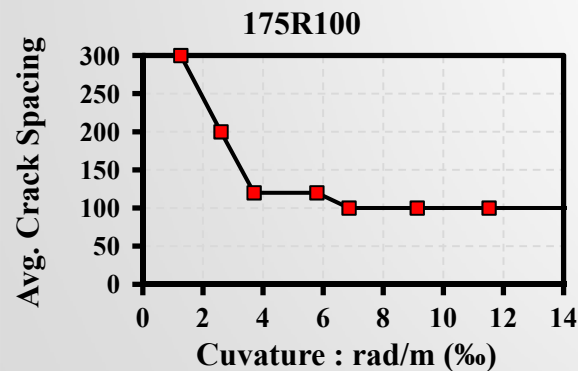
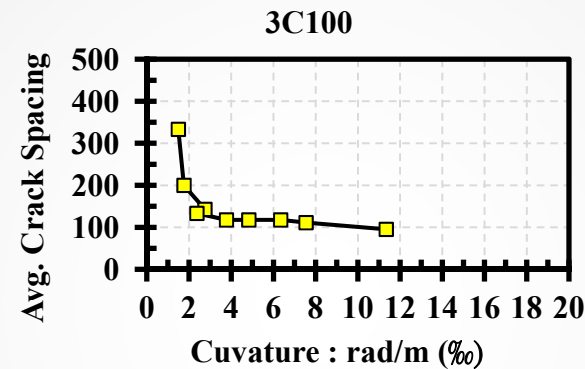
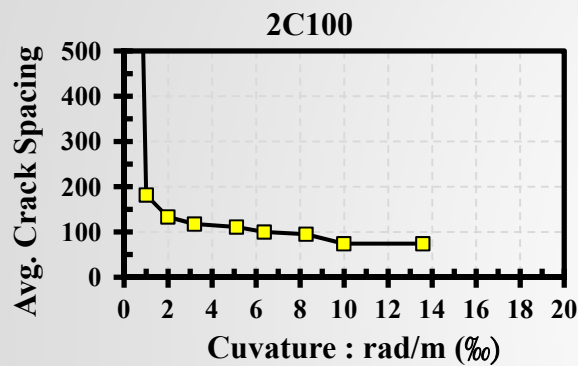
$$n_{f_Maximum} = 4.23$$

Crack width is proportional to reinforcement stress

Experimental Data

Flexural Crack

Stable → Crack number not increased → Crack spacing turns to be constant



Average crack spacing = Crack numbers / Span of the moment-equivalent region

Average Crack Spacing

Crack width = (strain)(crack spacing)

ACI 318 (2002 & 2005) $w_{max} = \frac{f_s}{E_s} \beta 2 \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$

AIJ-2010 (附錄) $w_{max} = 1.5(\epsilon_{s,av} + \epsilon_{sh}) \left\{ 2 \left(c + \frac{s}{10} \right) + k \frac{\phi}{p_e} \right\}$

JSCE-2007 $w_{max} = \left[\frac{\sigma_{se}}{E_s} + \epsilon'_{csd} \right] 1.1 k_1 k_2 k_3 \{ 4c + 0.7(c_s - \phi) \}$

CEB-FIP $w_{max} = (\epsilon_{sm} - \epsilon_{cm}) \left(k_3 c + \frac{k_1 k_2 k_4 \phi}{\rho_{p,eff}} \right)$

Broms $w_{max} = 1.66 \frac{f_s}{E_s} 2d_c$

趙唯堅與丸山 $w_{max} = 1.5 \left\{ \frac{\sigma_s}{E_s} - \frac{\sigma_{ctm}}{E_s p_e} - \epsilon_\phi \right\} \frac{1}{\alpha} k_1 k_2 (4c + 0.7e)$

Experimental Data

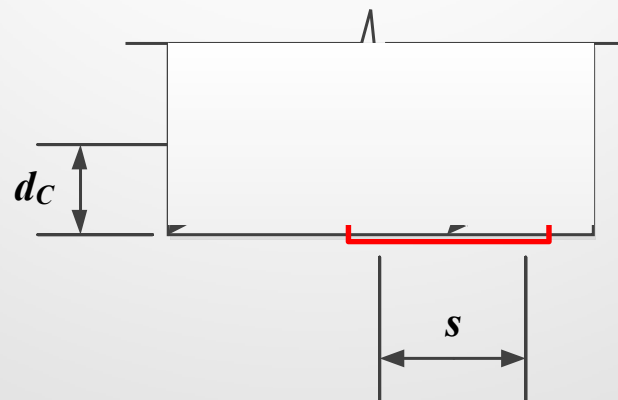
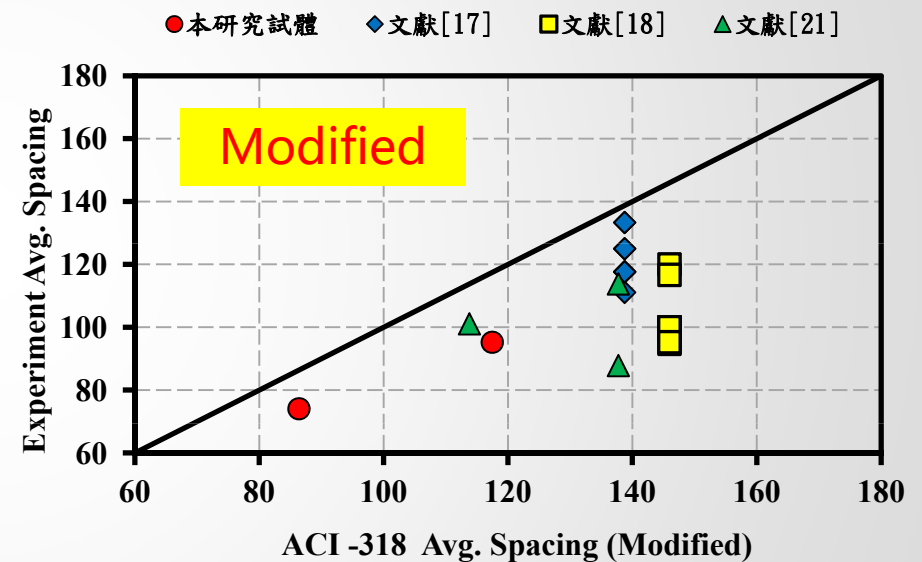
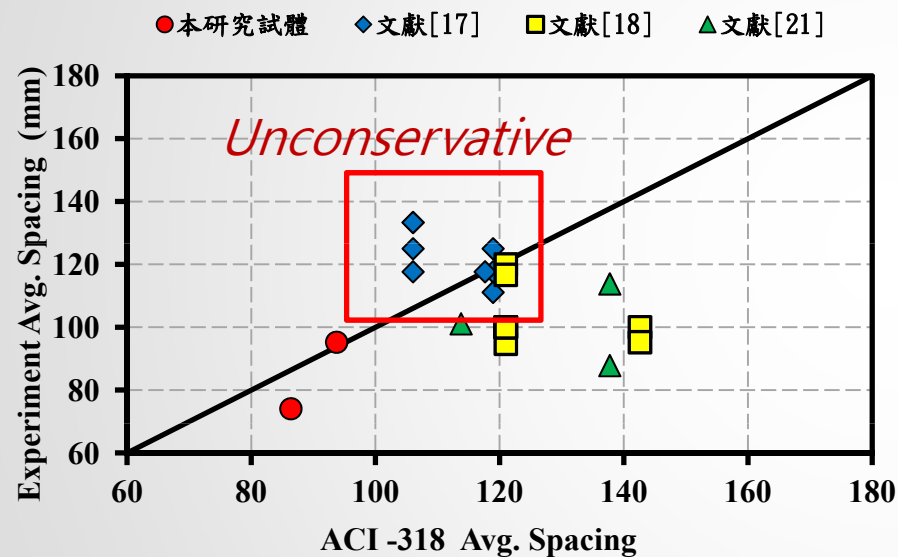
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Flexural Crack-Average Crack Spacing

$$\text{ACI-318} : \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$



$$\text{Max}[d_1^*, d_2^*]$$



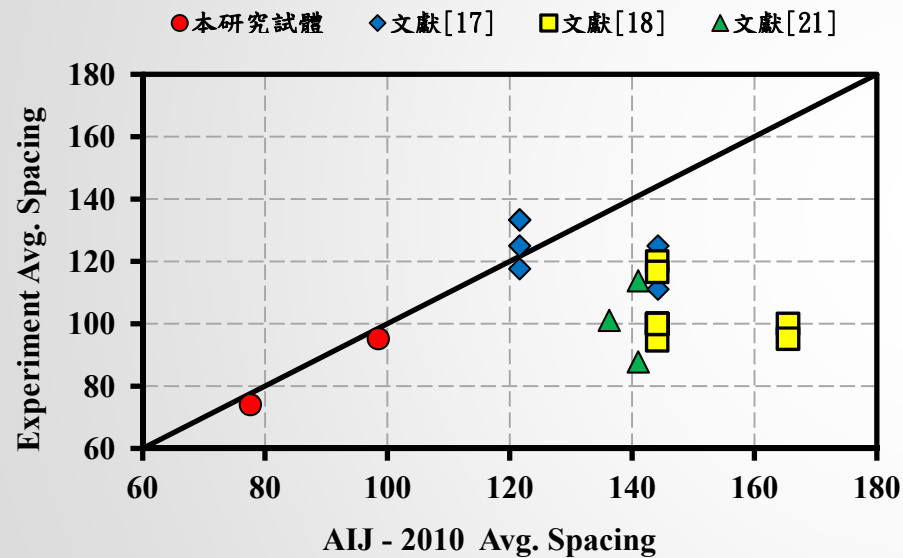
Experimental Data

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Flexural Crack-Average Crack Spacing

AIJ-2010 (附錄)

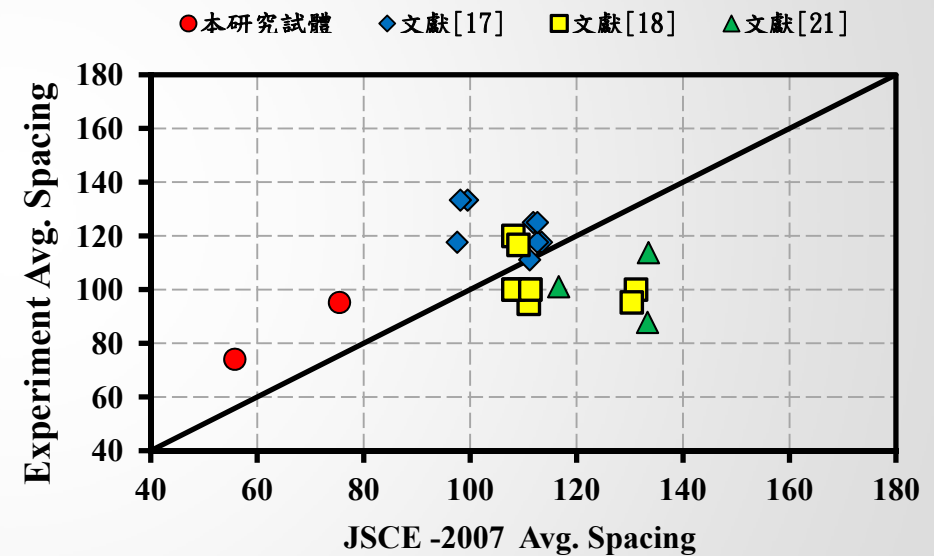
JSCE-2007



$$V_c = \frac{(V_{u,100\%} - V_{sc})}{(1.0 - 0.3)} \times (W_{p,max} - 0.3) + V_{sc}$$

$$V_s = V - V_c = \rho_{ts} f_s b d$$

$$f_{st} = \frac{V - V_c}{\rho_{ts} b d}$$



$$1.1k_1k_2k_3\{4c + 0.7(c_s - \phi)\}$$

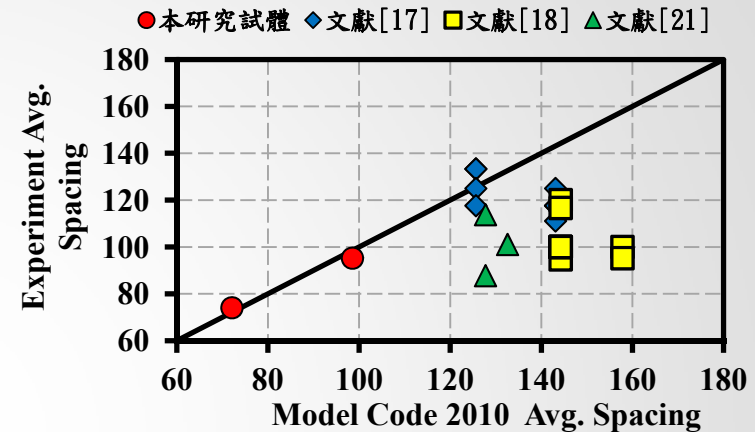
Experimental Data

Flexural Crack-Average Crack Spacing

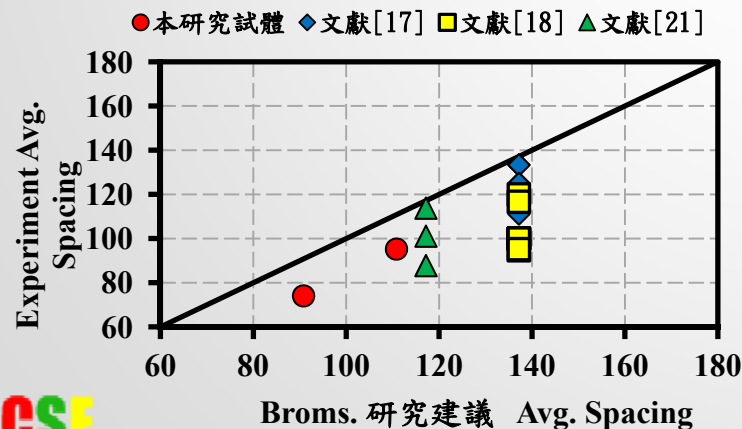
TAIWAN TECH

CEB-FIP

$$(k_3c + \frac{k_1k_2k_4\phi}{\rho_{p,eff}})$$

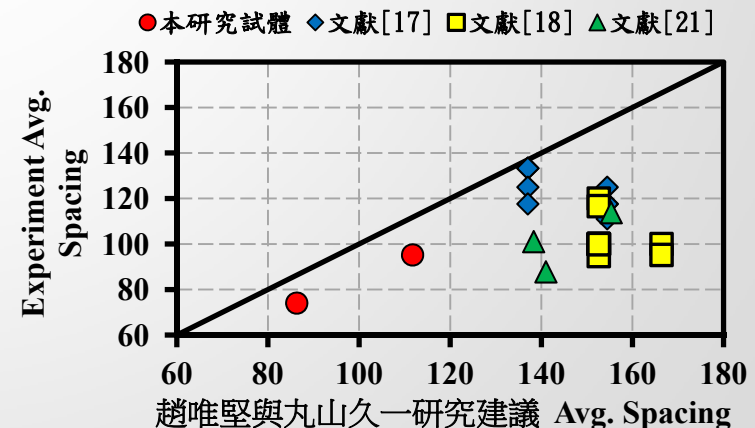


Broms $2d_c$



LCSE

趙唯堅與丸山 $\frac{1}{\alpha}k_1k_2(4c + 0.7e)$

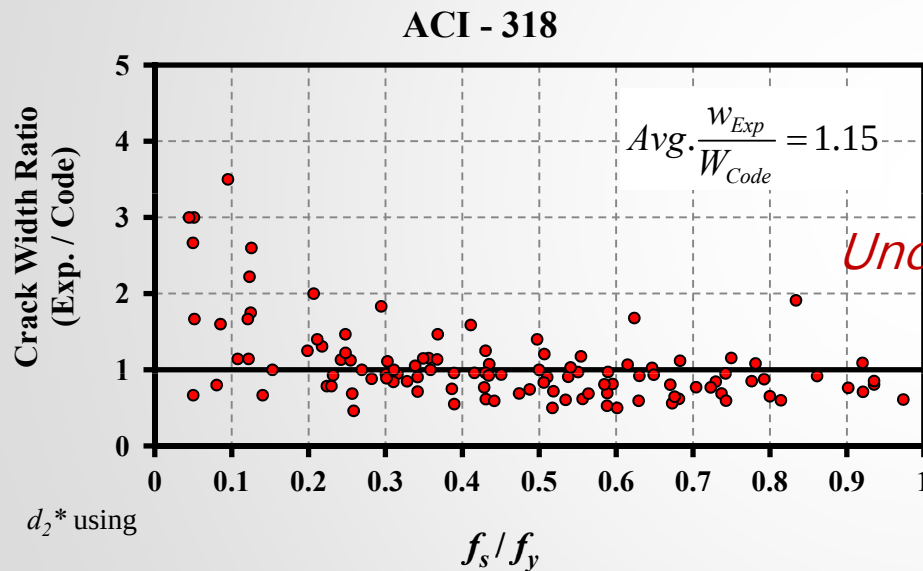


Experimental Data

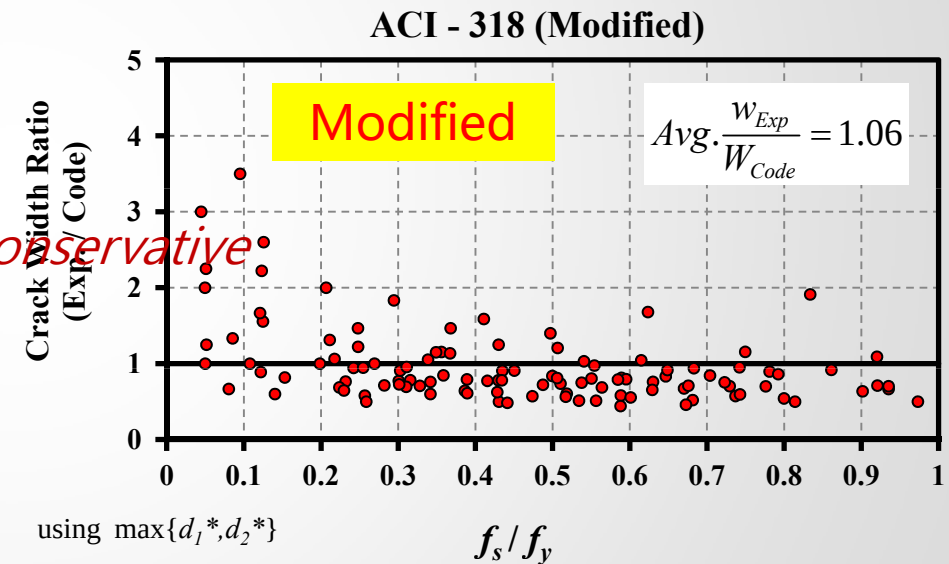
TAIWAN TECH

Flexural Crack Width

ACI 318 (2002 & 2005)



$$w_{max} = \frac{f_s}{E_s} \beta \sqrt{2} \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$



$$w_{max} = \frac{f_s}{E_s} \beta \sqrt{2} \times \max[d_1^*, d_2^*]$$

Experimental Data

TAIWAN TECH

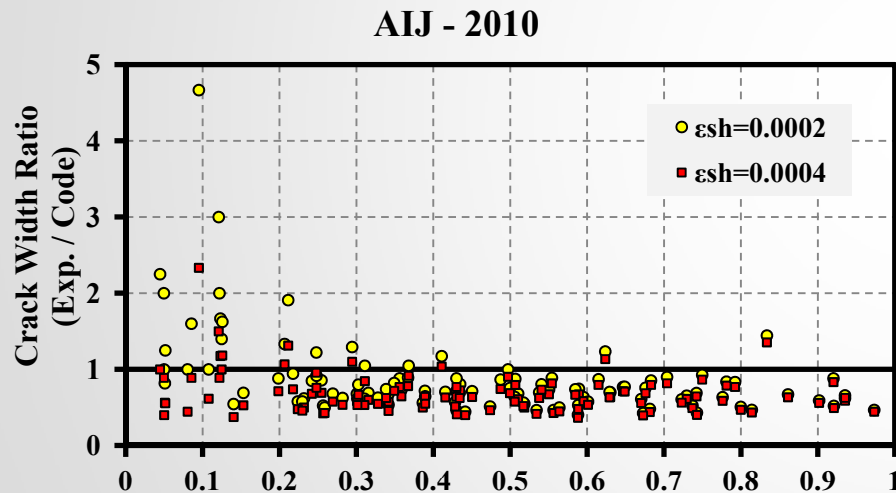
Flexural Crack-Average Crack Spacing

AIJ-2010(附錄)

$$w_{max} = 1.5(\epsilon_{s,av} + \epsilon_{sh}) \left\{ 2 \left(c + \frac{s}{10} \right) + k \frac{\phi}{p_e} \right\}$$

JSCE-2007

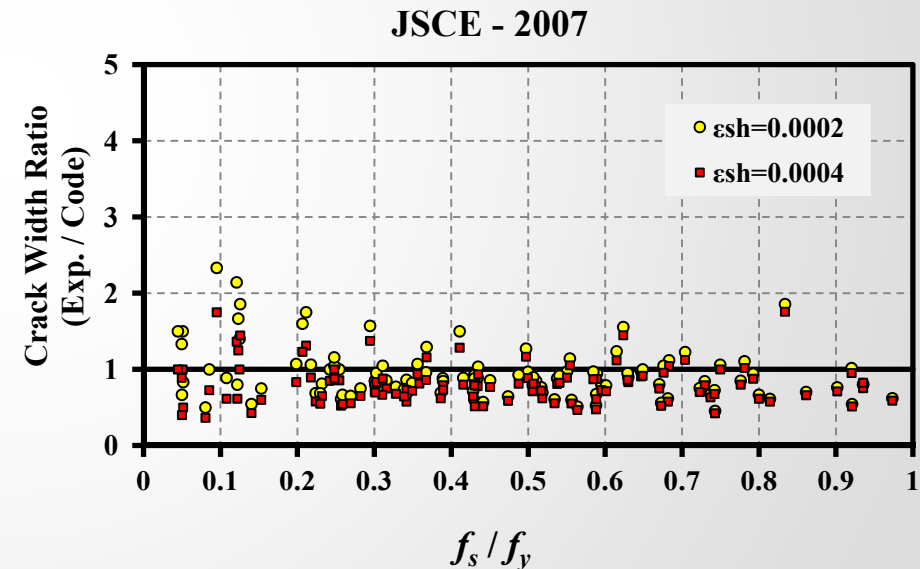
$$w_{max} = \left[\frac{\sigma_{se}}{E_s} + \epsilon'_{csd} \right] 1.1k_1k_2k_3\{4c + 0.7(c_s - \phi)\}$$



$$\epsilon_{sh} = 0.002, \text{Avg.} \frac{w_{Exp}}{w_{Code}} = 0.87$$

$$\epsilon_{sh} = 0.004, \text{Avg.} \frac{w_{Exp}}{w_{Code}} = 0.68$$

Good prediction
generally



Good prediction in low stress

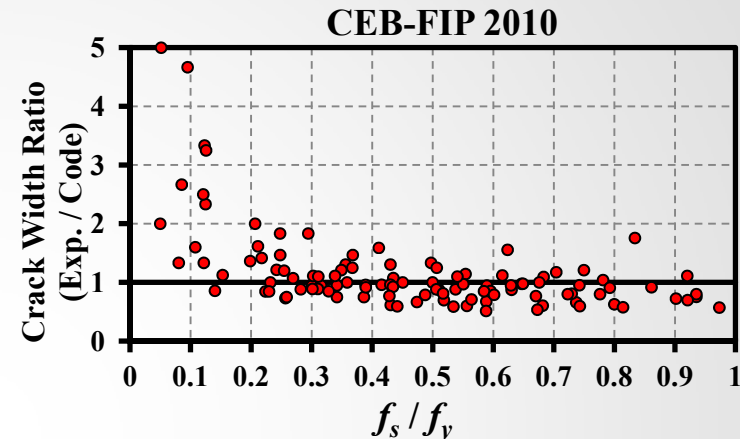
Experimental Data

TAIWAN TECH

Flexural Crack-Average Crack Spacing

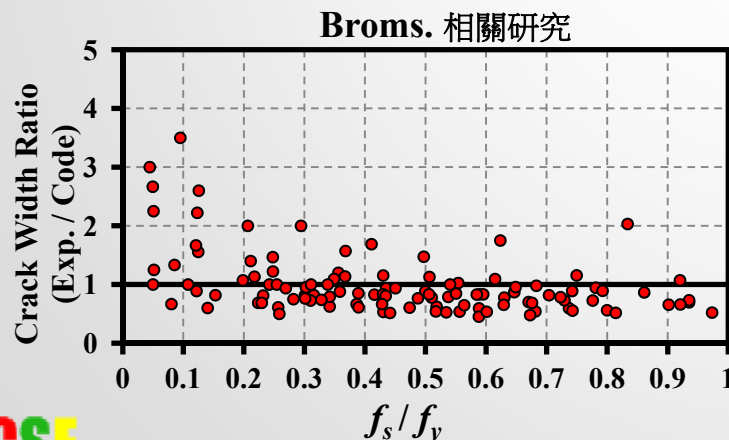
CEB-FIP

$$w_{max} = (\epsilon_{sm} - \epsilon_{cm})(k_3c + \frac{k_1k_2k_4\phi}{\rho_{p,eff}})$$



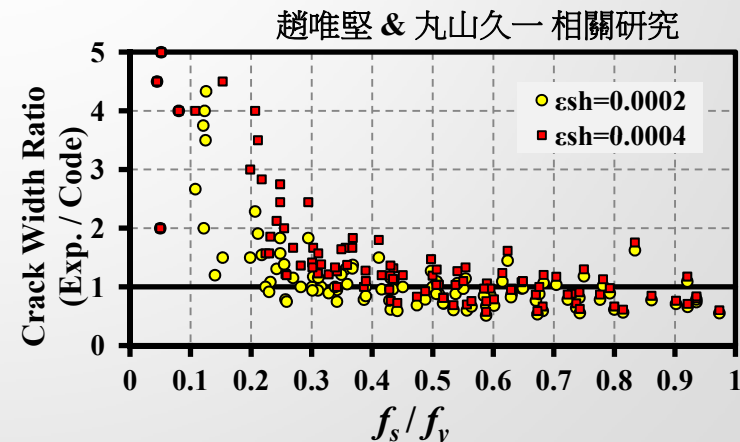
Broms

$$w_{max} = 1.66 \frac{f_s}{E_s} 2d_c$$



趙唯堅與丸山

$$w_{max} = 1.5 \left\{ \frac{\sigma_s}{E_s} - \frac{\sigma_{ctm}}{E_s \rho_e} - \epsilon_\phi \right\} \frac{1}{\alpha} k_1 k_2 (4c + 0.7e)$$



Flexural Crack Control

TAIWAN TECH

Design Formula

Problems :

- (1) Low stress level
- (2) Section parameter

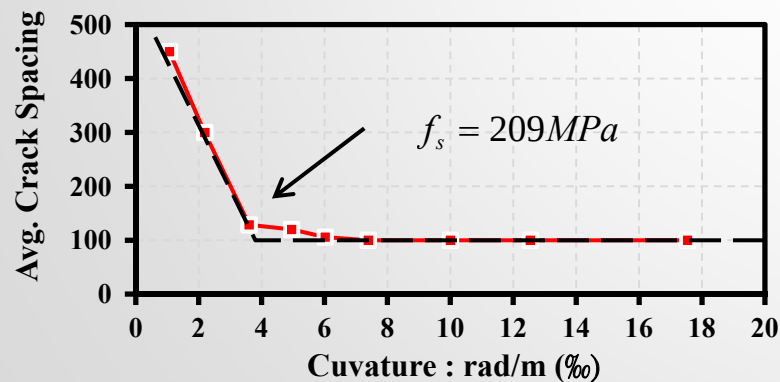
Improving : Crack spacing

Assumption :

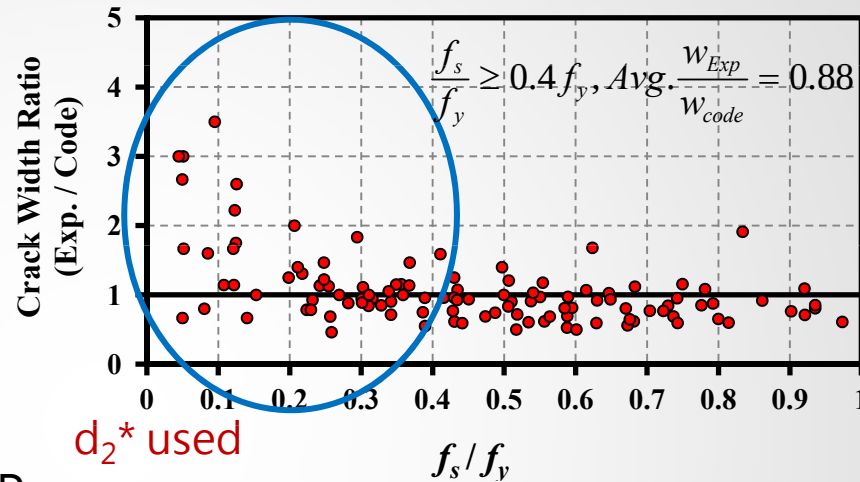
Bi-linear

Average stress : 263MPa

275R100

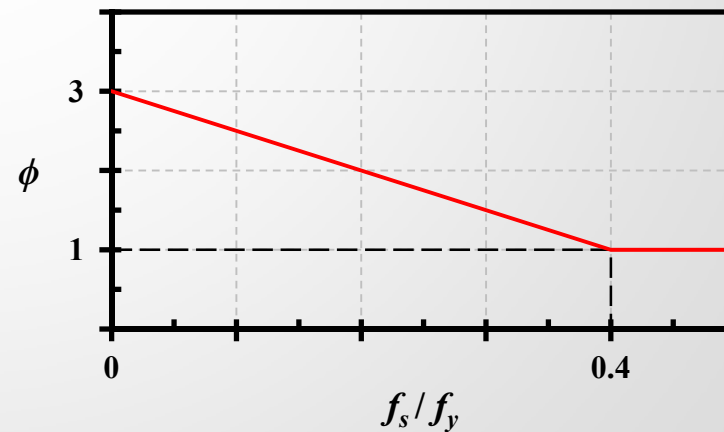


ACI - 318



d_2^* used

$$\phi = 3 - 5 \left(\frac{f_s}{f_y} \right), 0 \leq f_s / f_y \leq 0.4$$



Flexural Crack Control

TAIWAN TECH

Design Formula

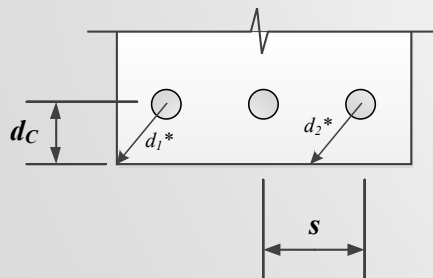
Problems :

(1) Low stress level

$$w_{max} = \phi \cdot 2\beta \frac{f_s}{E_s} \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

$$\phi = 3 - 5(f_s / f_y), 0 \leq f_s / f_y \leq 0.4$$

(2) Section parameter



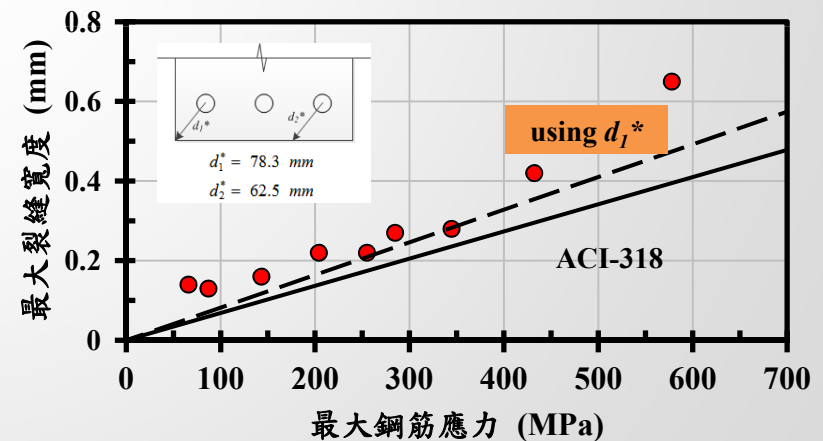
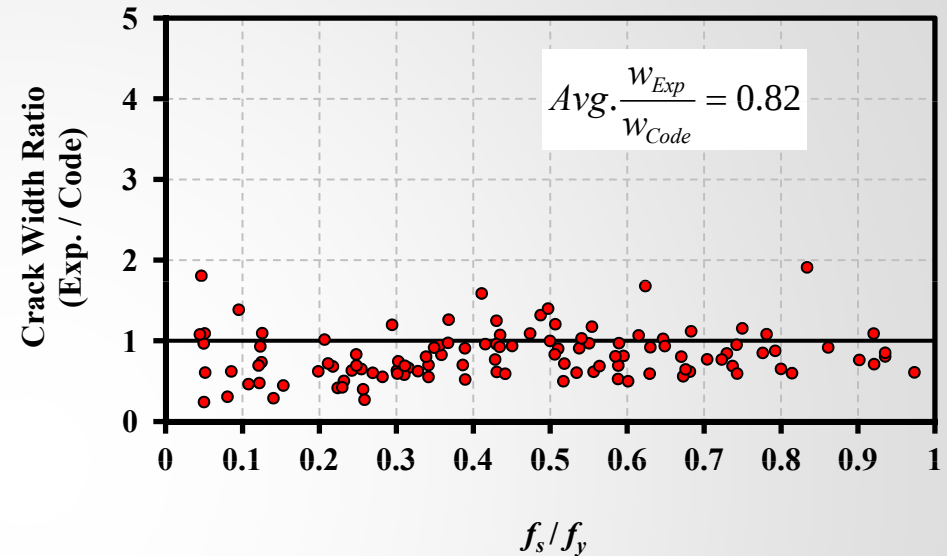
$d_2^* \leq d_1^*$: 預測偏低

$$d_2^* \geq d_1^*$$

$$\sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \geq \sqrt{d_c^2 + d_c^2}$$

$$s \geq 2d_c$$

Modified



Flexural Crack Control

TAIWAN TECH

Design Formula

$$W_{max} = \phi \cdot 2\beta \frac{f_s}{E_s} \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

$$f_s \geq 0.4f_y$$

Proposed formula

$$y = 98.79x$$

$$s \leq 280 \frac{274}{f_s}$$

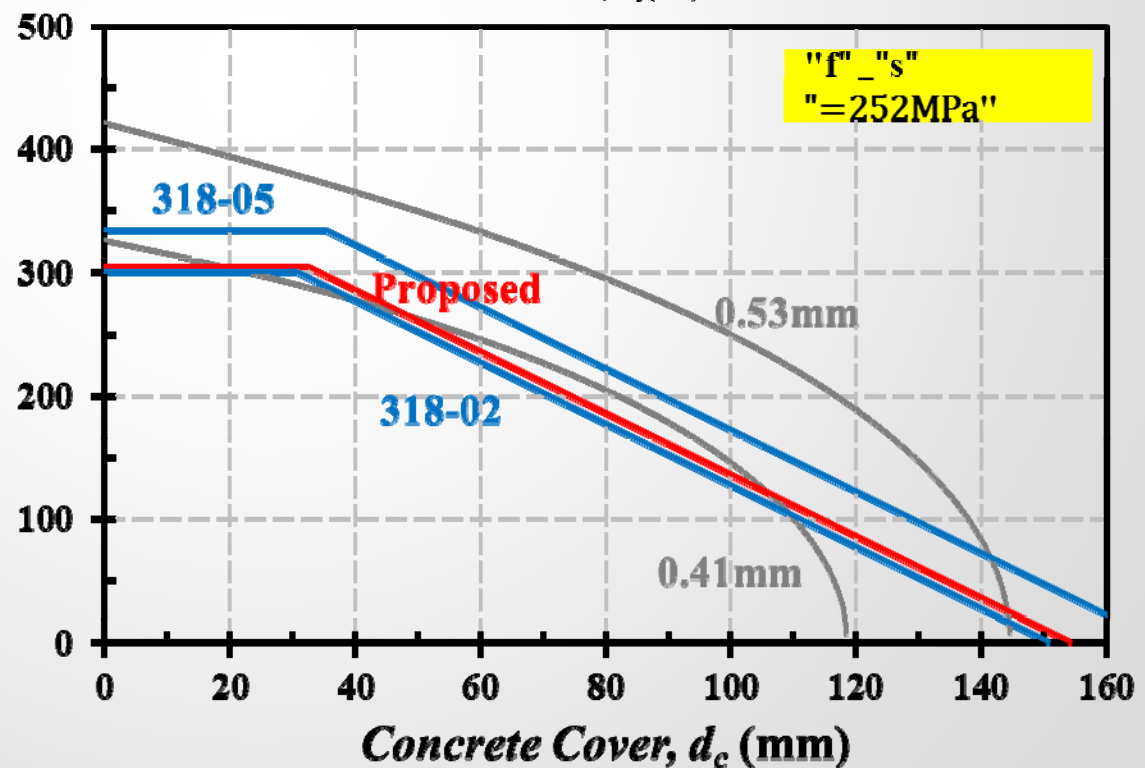
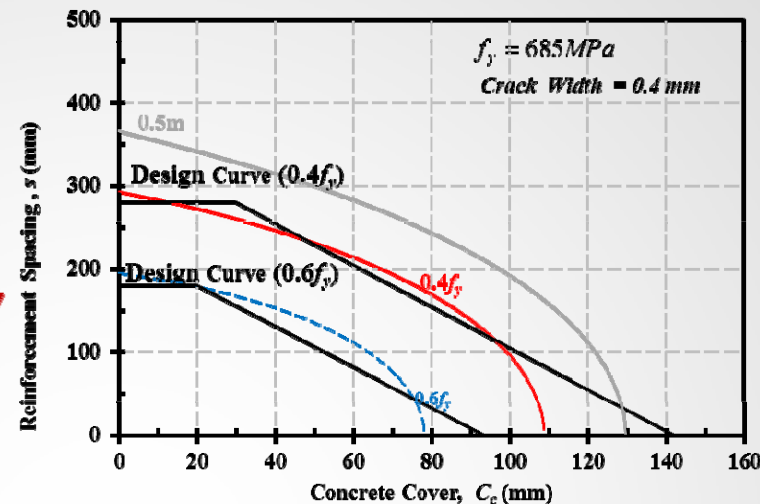
$$s \leq 380 \frac{280}{f_s} - 2.5c_c$$

$$s \leq 300 \frac{280}{f_s}$$

LCSE

$$f_s \geq 0.4f_y$$

Reinforcement Spacing, s (mm)



Flexural Crack Control

TAIWAN TECH

手冊建議, 4.3.2

$$s \leq 380 \frac{280}{f_s} - 2.5C_c \quad s \leq \frac{300 \cdot 280}{f_s}$$

f_s 為在使用載重下算得之鋼筋應力，若其小於 $0.4f_y$ 時，須以 $0.4f_y$ 取代之

[解說] 依現階段之實驗結果可知，梁撓曲裂縫控制可延用ACI318之規定。然而於使用載重下梁之鋼筋應力偏低，則依式計算所得之撓曲裂縫間距會有過小之疑慮，而造成裂縫寬度之計算值亦偏小而不保守；因此， f_s 若其小於 $0.4f_y$ 時，須以 $0.4f_y$ 代入式，以決定鋼筋中心距 s 之最大容許值[4]。

[解說] 依ACI318-02可知，式之成立條件為假設鋼筋中心距 s 之一半會大於撓曲受拉鋼筋中心至最近受拉面之距離，且依現階段之實驗結果可知，若鋼筋中心距 s 之一半小於撓曲受拉鋼筋中心至最近受拉面之距離時，則依式計算所得之撓曲裂縫間距會有過小之疑慮；因此，本手冊建議鋼筋中心距 s 之選取不宜小於 $2C_c$ [4]。

- Shear Crack Control

Allowable Shear force

$$V_{ser} = \left(\eta_{cr} \times \frac{0.33\sqrt{f'_c}}{1.5} + 0.15\rho_{ts}f_{yt} \right) bd$$

$$V_{rep} = \left(0.27\eta_{cu} \times 0.33\sqrt{f'_c} + 0.20\rho_{ts}f_{yt} \right) bd$$

- Flexural Crack Control

Spacing of reinforcement

$$s \leq 380\left(\frac{280}{f_s}\right) - 2.5c_c$$

$$s \leq 300\left(\frac{280}{f_s}\right)$$

$$f_s \geq 0.4f_y$$

Thank you



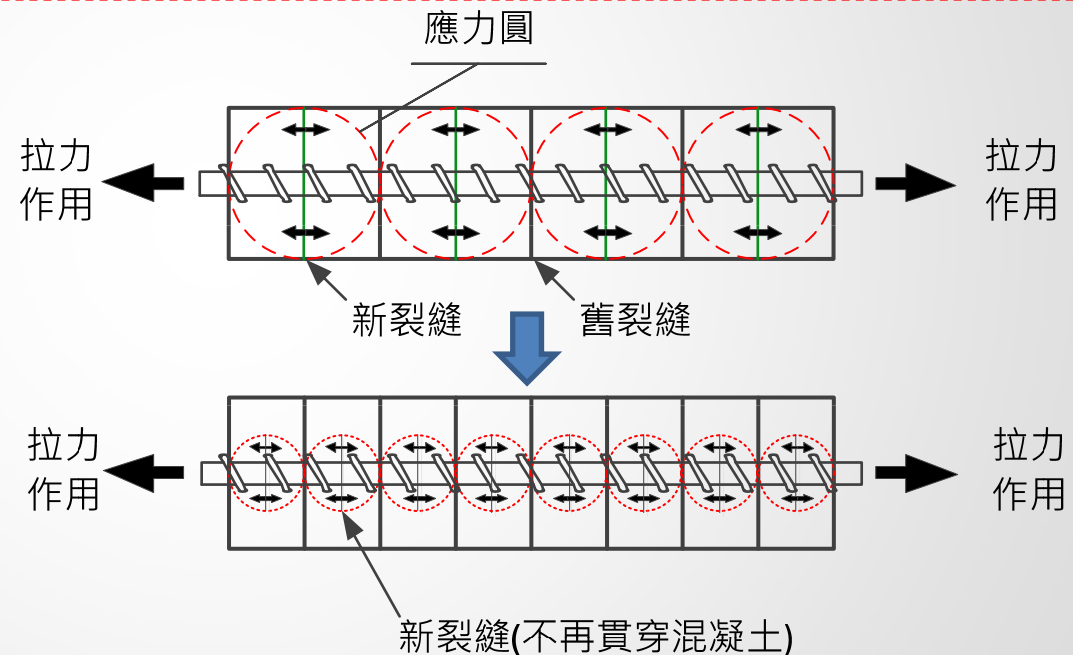
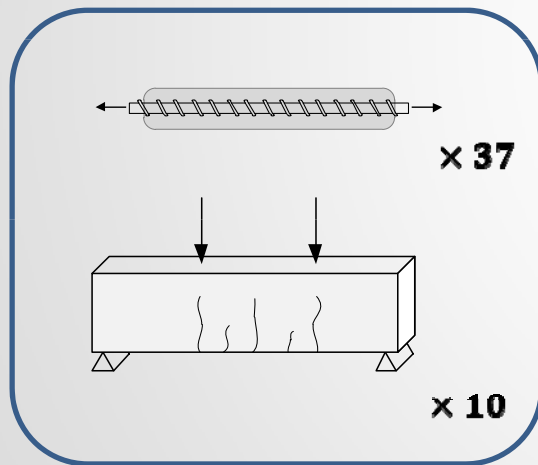
**Life-Cycle of
Structure Engineering**

Past Researches

Flexural Crack

Bengt. B. Broms (1965)

Specimens



1. Stable state : 140~210MPa
2. Crack spacing : $2d_c$
3. Crack width is proportional to the steel stress

文獻回顧

撓曲裂縫發展

各國撓曲裂縫控制規範

剪力裂縫控制

損傷評估

美國ACI-318規範

歐洲EuroCode 2規範

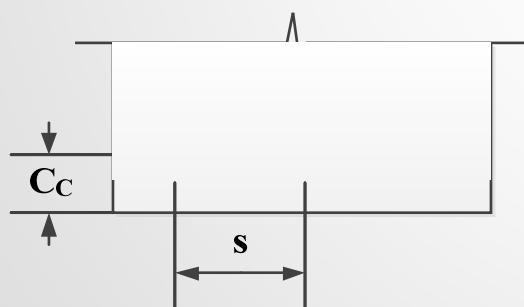
日本AIJ-2010附錄

日本JSCE-2007

$$s \leq 380 \left(\frac{280}{f_s} \right) - 2.5C_c$$

$$s \leq 300 \frac{280}{f_s}$$

裂縫寬度：0.4mm



不計算裂縫寬度

(1)滿足最小鋼筋量
(2)應力限制
(3)限制鋼筋直徑及間距

$$A_{s,min} \sigma_s = k_c k f_{ct,eff} A_{ct}$$

$$f_s \leq 0.75 f_y$$

$$f_c \leq 0.6 f'_c$$

鋼筋應力 (MPa)	鋼筋最大間距 (mm)			鋼筋應力 (MPa)	鋼筋最大直徑 (mm)		
	$w_k=0.4\text{mm}$	$w_k=0.3\text{mm}$	$w_k=0.2\text{mm}$		$w_k=0.4\text{mm}$	$w_k=0.3\text{mm}$	$w_k=0.2\text{mm}$
160	300	300	200	160	40	32	25
220	300	250	150	220	32	25	16
240	250	200	100	240	20	16	12
280	200	150	50	280	16	12	8
320	150	100	-	320	12	10	6
360	100	50	-	360	10	8	5
				400	8	6	4
				450	6	5	36

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裂縫寬度0.3~0.4mm

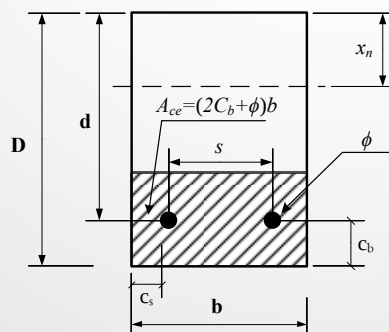
鋼筋型號	長期容許應力	
	抗壓及抗拉	抗剪
SD295	195	195
SD345	215(*195)	195
SD390	215(*195)	195
SD490	215(*195)	195
SD785	580	580
*D29以上之鋼筋為()內數值		

附錄

$$w_{max} = l_{av}\epsilon_{s,av} + l_{av}\epsilon_{sh}$$

$$l_{av} = 2\left(c + \frac{s}{10}\right) + k \frac{\phi}{p_e}$$

可適用混凝土抗壓強度
60~100MPa



鋼筋混凝土標準規範

$$w = 1.1k_1k_2k_3\{4c + 0.7(c_s - \phi)\} \times \left[\frac{\sigma_{se}}{E_s} + \epsilon'_{csd}\right]$$

裂縫寬度限制 $\leq 0.3mm$

$$k_2 = \frac{15}{f'_c + 20} + 0.7 \leq 0.9$$